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Determination of the Stress Distributions in a Ceramic, Tensile Specimen Using Numerical Techniques*

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ABSTRACT

Finite element analyses (FEA) were used to determine the stress distributions in a ceramic, tensile specimen with two types of button-head gripping systems. The FEA revealed stress raisers at both the button-head and the transition from the gage section to the shank. However, the stress field within the bulk of the gage section is uniform and uniaxial. The stress ratio, k_t, between the button-head and gage section stresses varied from 0.35 to 0.72 for the tapered collet or the straight collet systems, respectively. Previous empirical tests confirm these results whereby, the tapered collet system, compared to the straight collet system, sustained over twice the average load before failure at the button-head.

NOMENCLATURE

- D - maximum cross sectional diameter of button head.
- E - elastic modulus.
- k_t - stress ratio of maximum, tensile, principal stress in button head to the uniform principal stress in the gage section $[\sigma_{11}^{bh}/\sigma_{11}^{gs}]$.
- K_t - stress concentration factor.
- l - total length of specimen.
- r - radius of gage section.
- R - radius of button-head/shank transition.
- x - radial direction from longitudinal axis.
- y - direction along longitudinal axis.
- ν - Poisson ratio.
- σ_{11}^{bh} - maximum, tensile principal stress in button head.
- σ_{11}^{gs} - uniform principal stress in the gage section.

INTRODUCTION

Sophisticated life prediction methodologies proposed for use in advanced ceramic designs (1,2) require as inputs, the mechanical properties of ceramics as measured in uniform, uniaxial stress fields. A popular and efficient means for obtaining uniform stress fields is through the use of uniaxially-loaded tensile specimens (3). Various specimen types and geometries have been proposed and successfully applied in tensile tests ranging from measurements of load-displacement curves (4,5) and monotonically- or cyclically-loaded strengths (5-11), to the determination of creep [stress-rupture] response at elevated temperatures (12-22).

The commonly-used button-head tensile specimen, analyzed in this study and shown in Figure 1, is a variation of a design which has been employed for several decades (3). Advantages of the cylindrical, button-head specimen include symmetrical loading, tendency toward uniform load-transfer to minimize bending, relatively simple gripping systems, large ratios of volume to surface area, and uncomplicated fabrication of the specimen (3,7-9,11-15). However, recent increases in the use of tensile tests for structural ceramics, coupled with increasing ultimate strengths in these materials, have revealed peculiarities in the locations and circumstances of failures in this type of

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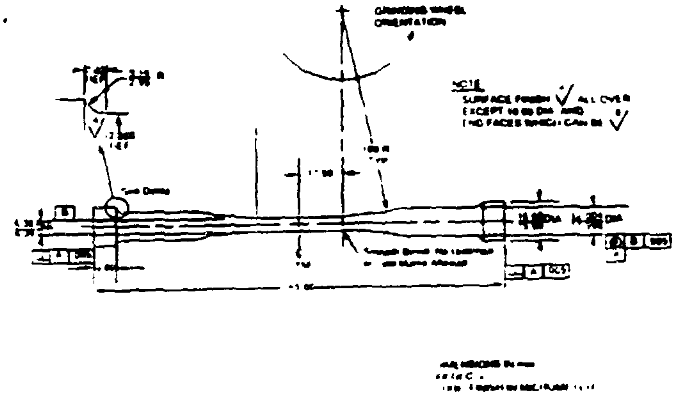

Figure 1 Cylindrical. Button-head. Tensile Specimen.

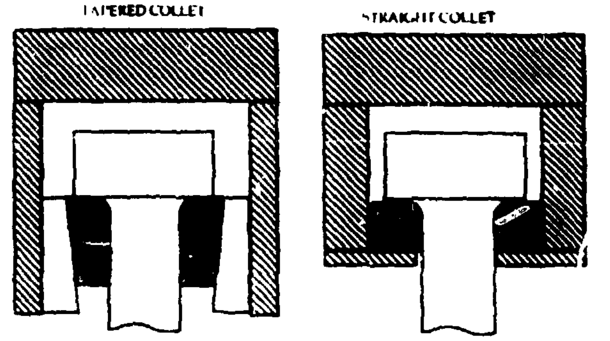

Figure 2. Comparison of Tapered Collet and Straight Collet Gripping Systems.

tensile specimen [e.g.. non-gage section fracture especially in the button-head area].

The objective of this study was to numerically determine the stress distributions in a ceramic. button-head tensile specimen. For verification. the numerical results were compared to empirical tensile tests conducted in conjunction with hydraulic couplers in the load train which reportedly hold percent bending to ~1.0 at specimen failure (8.23).

SYSTEMS FOR GRIPPING THE SPECIMEN

Numerous gripping systems for the button-head specimen have been described in previous studies (3.7-9.11-18). However. only two types of grip arrangements. as shown in Figure 2. were examined in this study due to their frequency of use compared to other systems. For each gripping system the overall specimen geometries and dimensions [Figure 1] were identical. differing only in the radius of the button-head. R: nominally. 3.0 mm [shown] and 2.38 mm for the straight and tapered collet systems. respectively.

The particular straight collet system examined in this study transfers the applied load directly into the root radius of the specimen button-head. Advantages of this system are i) direct loading from the grip into a consistent part of the specimen and ii) the relatively small area at the specimen/grip interface which reduces the area requiring critical tolerances. Disadvantages are i) the reliance on the button-head to carry the complete applied load and ii) superposition of the direct and/or frictional loading on the inherent stress concentration of the button-head radius.

The tapered collet system transfers the applied load. through friction enhanced by lateral compression. primarily into the shank of the specimen. The obvious advantage of this system is that the button-head is not the primary load-bearing part of the specimen thus the criticality of machining damage or inherent material flaws in the root radius area is reduced. However. less obvious disadvantages are i) the necessity of maintaining close dimensional tolerances over the larger area of the shank. ii) the possibility of collet jamming/mismatch leading to eccentric loading [and bending]. and iii) the variability of the coefficient of friction at the specimen/collet interface which may change with surface preparation or testing conditions.

FINITE ELEMENT ANALYSES

Recently. a number of numerical approaches have been undertaken to understand the stress state in the button-head region of the specimen design illustrated in Figure 1. Work conducted in the present study will be discussed followed by discussions of efforts conducted at Garrett Auxiliary Power Division [GAPD](25) and the University of Dayton Research Institute [UDRI](23).

Present Study

FEA techniques were applied to ascertain the stress distributions in the specimen as influenced by the straight and tapered collet gripping systems. The objective of the study was to investigate the interaction between the collets and the specimen as well as to identify key parameters such as friction at the collet/specimen interfaces which might influence the stress distributions in the button-head area. COSMOS/M. a commercial PC-based finite element code (26) running in the protected mode of the Intel 80386 processor. was used in conjunction with the sophisticated geometric modeler. GEOSTAR (26). to perform the analysis of the specimen and the gripping systems.

The axi-symmetric [y-axis along the longitudinal axis of the specimen]. quarter-symmetry models were composed of ~7500-9500 degrees-of-freedom [DOF]. Two dimensional. four-noded plane elements. were used to model the structure of the specimen and collets. Non-linear. frictional. "gap" elements (26) were used to model the interfaces between the specimen and collet at both the button-head root radius and the shank. Since these particular "gap" elements did not contribute to the overall stiffness matrix of the specimen/collet system: ultra-low stiffness, two-dimensional truss elements were used to provide remote, mathematical constraints to the collet in the necessary directions. Specimen dimensions were those as shown in Figure 1 except for the .utton-head radius which was changed to match the particular gripping system as previously discussed.

For the specimen, material-symmetry boundary conditions [BC] were applied along the longitudinal axis and at the "free" end of the gage section. These BC's were modeled as rollers where free, nodal displacements were allowed parallel to the surface but the nodal displacements were constrained normal to the surface. Similar BC's were applied to the collet systems to simulate the constraints of the gripping arrangement.

The linear-elastic material properties of the specimen were those of an isotropic silicon nitride at room temperature with an elastic modulus, E=310 GPa and a Poisson ratio, ν=0.27. The properties of the collets were those of steel in which E=200 GPa and ν=0.3. As noted, the truss elements were used only for mathematical constraint, thus E=1×10^{-6} GPa and ν=0.3.

Due to the non-linear behaviour of the gap elements, the element pressure loading on the collets was applied in incremental steps allowing structural equilibrium to be reached at each step by an iterative process. The size and number of time steps, as well as the refinement of the element mesh, was determined manually through a trial-and-error method of examining the convergence of the model displacements.

The two models, which include the specimen and gripping systems, are shown in Figures 3 and 4. Figures 5 and 6 illustrate the variation of the normalized maximum, tensile, principal stresses $[\sigma_{11}/\sigma_{11}^{gs}]$ as a function of normalized, longitudinal distance [y/(l/2)] from the center of the specimen.

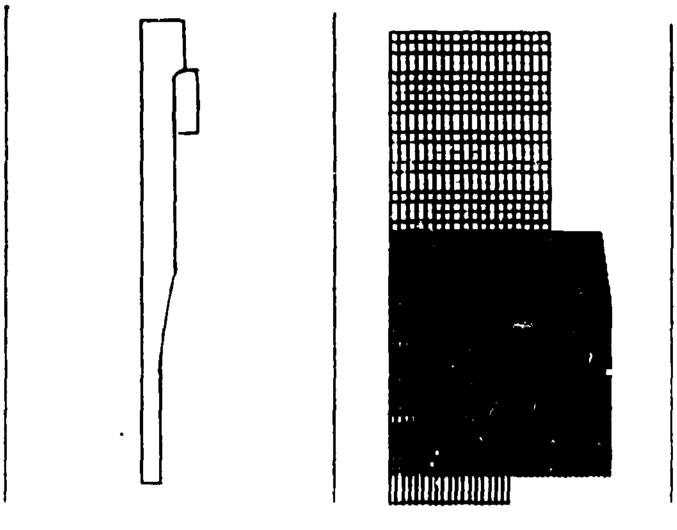

Figure 3. Straight Collet FEA Model for the Present Study.

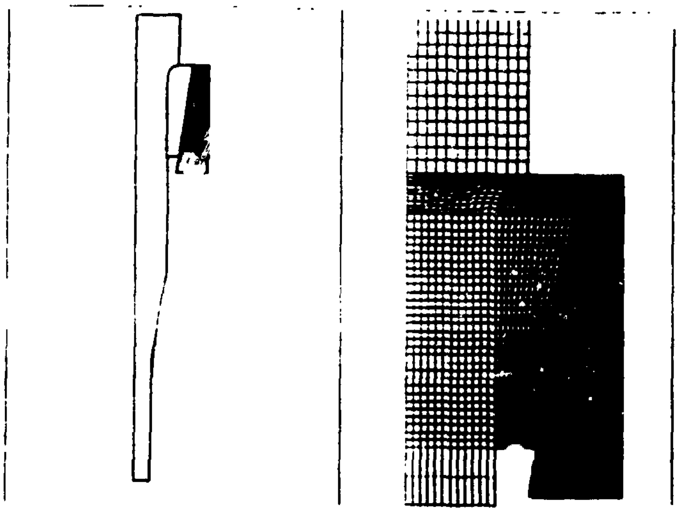

Figure 4. Tapered Collet FEA Model for the Present Study.

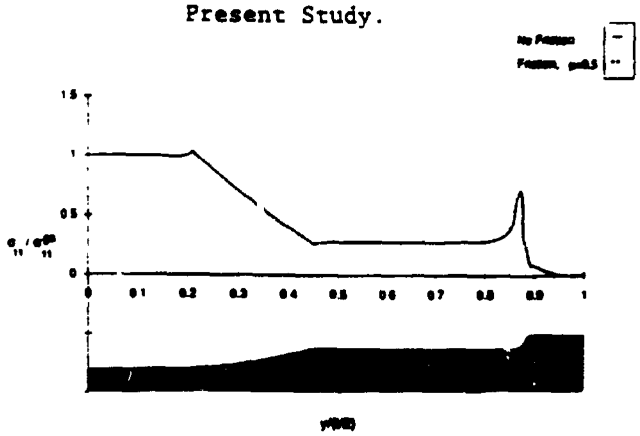

Figure 5. Normalized, Tensile, Principal Stresses for a Straight Collet System in the Present Study.

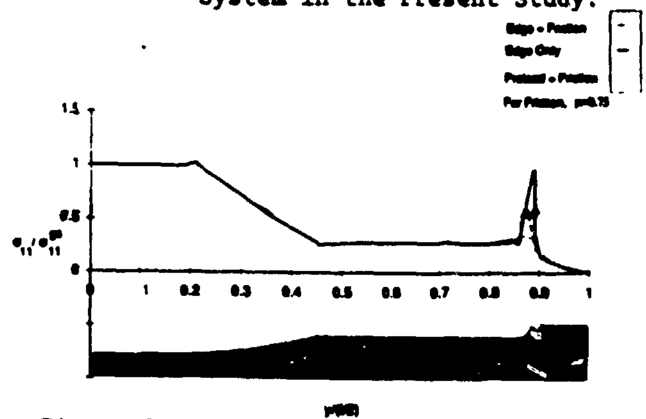

Figure 6. Normalized, Tensile, Principal Stresses for a Tapered Collet Systems in the Present Study.

Two anomalies in the stress distributions should be noted in Figures 5 and 6. The first is that, for both gripping systems, the uniform, uniaxial stress state in the gage section [0<y/(l/2)<0.212] is perturbed as the gage section begins the transition into the large radius leading to the shank. This perturbation, illustrated in Figure 7, results in a surface stress riser which is ~5% greater

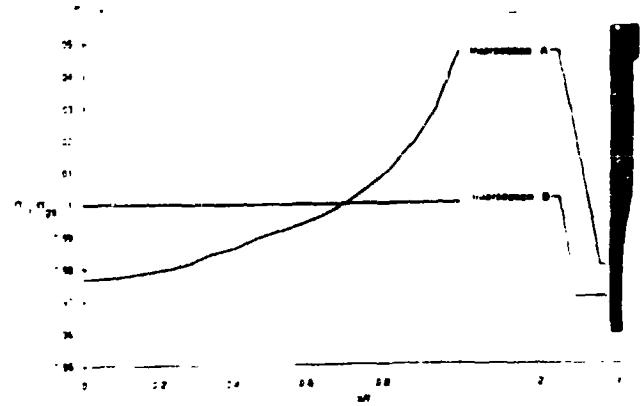

Figure 7. Typical Normalized. Tensile. Principal Stresses across the Gage Section for Both Gripping Systems in the Present Study.

than the uniaxial. gage section stress. Thus. for a uniaxially-aligned testing system and a defect-free material. the distribution of the locations of gage-section failures may tend to skew towards the surface of this transition.

The second stress anomaly is in the area of the button-head radius. In Figure 5 for the straight collet system. the stress ratio. k_t [i.e. the ratio of the maximum. tensile. principal stresses where $k_t = \sigma_{11}^{bh}/\sigma_{11}^{gs}$] between the button head and the gage section is ~0.72 when a coefficient of friction. μ=0.5. is used for the collet/specimen interface. For μ=0.0 [frictionless]. k_t=0.69. indicating that friction may not be as critical a contribution to the stress state as the inherent stress concentration of the geometry and the loading condition of the straight collet system. In Figure 6. for the tapered collet system. k_t ranges from 0.59 to 0.97. if the collets contact the outer edge of the button-head from the start of the loading sequence. with or without friction. respectively. This simulates the installation of the collets with no regard to preloading the collet against the specimen shank. If a preload is simulated at the collet/specimen interface. k_t=0.35 since a greater part of the load is transferred directly into the shank.

The conclusions from this FEA study were:

a) the stress distribution in the gage section is for the most part uniform and uniaxial except for a small [~5%] stress riser near the transition into the shank.

b) the stress concentration in the button-head radius of the contacting straight collet system may cause the stress in the button-head region and the gage section of the material.

c) the tapered collet system can significantly reduce the stresses in the button-head region if a sufficient preload is applied before testing to prevent movement of the collets in relation to the specimen.

Garrett Auxiliary Power Division

The FEA modeling at GAPD (24) was used to ascertain the parameters influencing the stress state in the button-head region so as to accommodate a redesign of the gripping system and the specimen button-head. A hybrid approach was used in which a linear FEA solution was first obtained for the collet/specimen model. Where contact stresses were of concern. an analytically derived solution (27) for the Hertzian-type contact stresses between two cylinders was then superposed on the FEA linear-elastic solutions to obtain the solution for the final stress state. A Control Data Corporation Cyber main-frame computer was used in conjunction with the commercial finite element code ANSYS (28).

Approximately 3?00 DOF were used in the axi-symmetric. quarter symmetry model of the gripping system and the specimen as shown in Figure 8. Two dimensional. isoparametric solid elements were used to form the structure. Essentially a parametric study was conducted to identify key dimensions or loading configurations which would minimize potential button-head failures. Maintaining the current 3.0 mm button-head radius and 6.35 mm diameter gage section. specific areas investigated were:

a) The effect of contact stresses on the button-head stress state for a straight-collet system. [Figure 9].

b) Development of a relationship between the stress ratio. k_t (defined earlier). and the button-head diameter [Figure 9].

c) Determination of optimum shank diameters for various button-head diameters [Figure 10].

d) Determine the effects of dimensional changes for the button-head length. the shank length. a double radius at the button-head.

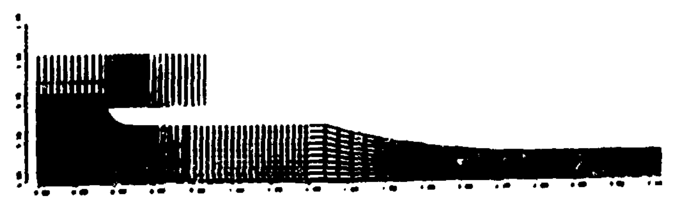

Figure 8. Straight Collet FEA Model for the GAPD study (24).

As shown in Figure 9. assumed contact stresses in the currently-used. straight collet system may cause k_t to approach 0.9 if the present button-head diameter is maintained. However. for increasing button-head diameters and/or the elimination of contact stresses. k_t can be decreased into the range of 0.35 to 0.60.

For various button-head diameters. "optimum" shank diameters can be found as shown in Figure 10. The present button-head diameter of 16 mm [0.63 in] sharply limits the

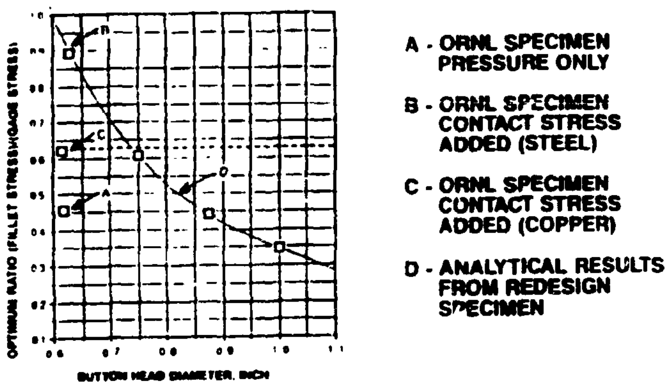

Figure 9. Effects of Contact Stress and Button-head Diameter on Normalized Stress in the GAPD study (24).

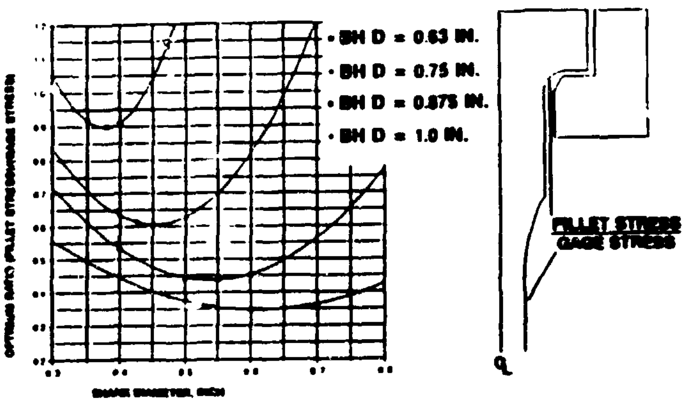

Figure 10. Optimum Shank Diameters for Various Button-head Diameters in the GAPD study (24).

choice of the shank diameter even for a relatively high k_t [k_t=0.9], while a button-head diameter, D, of 25.4 mm [1.0 in] allows a wider choice of shank diameters for an acceptable k_t [0.35 to 0.40].

For the range of specimen dimensions examined, small effects on k_t were found for dimensional changes in the button-head length, shank length, and a double radius at the button-head. The conclusions of the GAPD study can be summarized as follows:

a) Contact stresses combined with the inherent stress concentration in the button-head region may cause unacceptably large k_t values.

b) The gripping system should be redesigned to eliminate contact stresses in the critical button-head region.

c) Acceptably low k_t values can be achieved with a non-frictional gripping system in combination with a button-head diameter of ~22.0 mm, a shank diameter of ~14.0 mm, and a button-head radius of ~3.0 mm.

University of Dayton Research Institute

Concurrent FEA modeling at UDRI (25) was aimed at determining the k_t effects over a range of loading situations in the button-head region. Various loading scenarios, as shown in Figure 11, were simulated using appropriate element pressures and a simple linear-elastic model of the button-head/shank portion of the specimen.

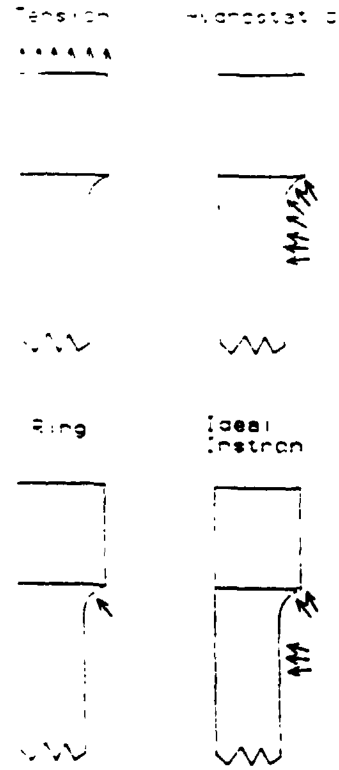

Figure 11. Loading Scenarios assumed in the UDRI study (25).

An axi-symmetric, quarter symmetry model was used with approximately 12000 to 13000 DOF as partially illustrated in Figure 12. Four noded, bi-linear elements (29) were used in the analysis which was conducted on the PC-based commercial code, SUPERSAP (29). The specimen material properties were those of silicon carbide [E=427 GPa, v=0.14] with the specimen dimensions as shown in Figure 1.

The tension case shown in Figure 11 was used to represent the ideal case of a unidirectional stress case and also served to validate the mesh geometry through comparison of the stress results with readily available analytical solutions. The k_t [as defined earlier] for the tension case was found to be 0.44. The general stress concentration factor at the button-head/shank transition as determined from the FEA model [K_t=1.57] compares well to the analytical case [K_t=1.40] (30) when it is realized that the FEA value is calculated from surface stresses and not axis stresses as is the analytical value.

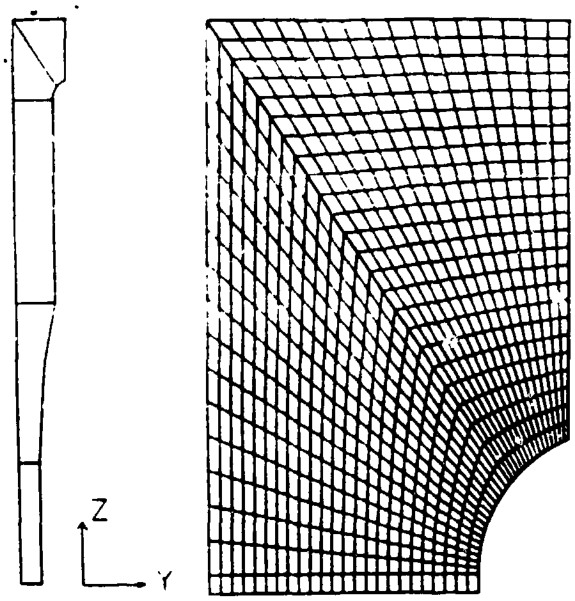

Figure 12. Straight Collet FEA Model in the UDRI study (25).

The hydrostatic pressure case of Figure 11 was intended to represent the case of perfect contact between a straight collet and the specimen. This situation may exist due to plastic deformation of some collet configuration [such as annealed-copper, straight collets (25) or BN powder (9,17)]. For this case, k_t=0.53, which is sufficiently low to explain successful [no button-head failures] tensile tests using "soft" collet systems (9,17,19,25).

The ring loading and 'ideal-Instron' (23) cases shown in Figure 11 were investigated to determine the effects of various scenarios for "hard" straight collets illustrated in Figure 2. The ring-loading would occur if there was a mismatch between the button-head and the collet radii. The k_t in this case is ~0.75 for mismatches of 1% to 10% (collet radius less than the button-head radius). For the ideal 'Instron' case, k_t=0.85 which is in the range of the Weibull strength of the material as mentioned earlier. It should be noted that the assumed loading for the UDRI ideal 'Instron' case did not agree with the loading observed in the FEA of the present study where the interactions between the collet and the specimen were actually modeled and investigated. Nevertheless, the k_t values for these idealized, linear-elastic cases are sufficiently high that one button-head failures might be expected for even minor Hertzian-type stresses in the contact areas.

The results of the UDRI study can be summarized as follows:

a) Hydrostatic loading provides "good" stress behaviour in the button-head region and can be realistically approached in the laboratory.

b) Increased contact area between the collet and the button-head can substantially reduce button-head failures.

c) Alternative geometries should be investigated to reduce the criticality of the inherent stress concentration at the button-head radius.

Summary of Finite Element Analysis Modeling

The three studies described here all took different approaches yet the results are in reasonable agreement. However, it is interesting to note the directions of the conclusions.

GAPD recommends the elimination of direct or frictional contact between the specimen and collet and advices enlarging the specimen dimension substantially to accommodate this change. However, both UDRI and the present study indicate the efficacy of direct, but conformable contact between the collet and button-head [deformable collets] or direct frictional contact between the collet and shank [tapered collets] which minimize the load-bearing role of the button-head.

Unfortunately, none of the studies addressed the equally important issue of the gripping system which is the minimization of bending strains. Ideally, the gripping system which ultimately eliminates non-gage section failures must also help to minimize the bending stresses in the gage section.

PREVIOUS EXPERIMENTAL TESTING

A series of tensile tests (32,33) was conducted on strain-gaged, 99% alumina,** button-head tensile specimens which did not have gage sections [straight-shank specimens]. Tests using self-aligning, hydraulic, load-train couplers (23) and the straight-shank specimens were primarily intended to determine the maximum loads which various configurations of the tapered and straight collet systems could sustain before specimen failure occurred in the gripped section [i.e. button-head or shank]. In addition, four, uniaxially-aligned strain gages were applied to each specimen. These gages were equi-spaced around the circumference at the mid-point of the specimen length such that the measurement of the percent bending (4,31) could be made throughout the loading sequence in order to determine the gripping system with the most potential for minimizing bending. The use of four strain gages provided the advantage of symmetry from which the plane of bending could be determined and cross-checks could be made of the uniaxial alignment of gages. The results of the experimental testing are summarized in Figures 13 and 14 which show the maximum load and percent bending at specimen failure, respectively. The standard configuration of the tapered collet system sustained a higher average load at button-head failure while maintaining a lower average percent bending as compared to all the configurations of the straight collet system.

**AD-995, Coors Porcelain Company, Golden, Colorado.

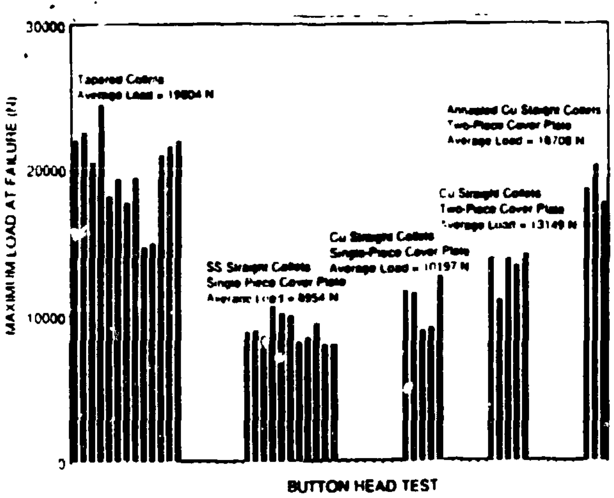

Figure 13. Maximum Load at Failure of the Gripped Area for Tensile Tests of Straight-shank Specimens(32.33).

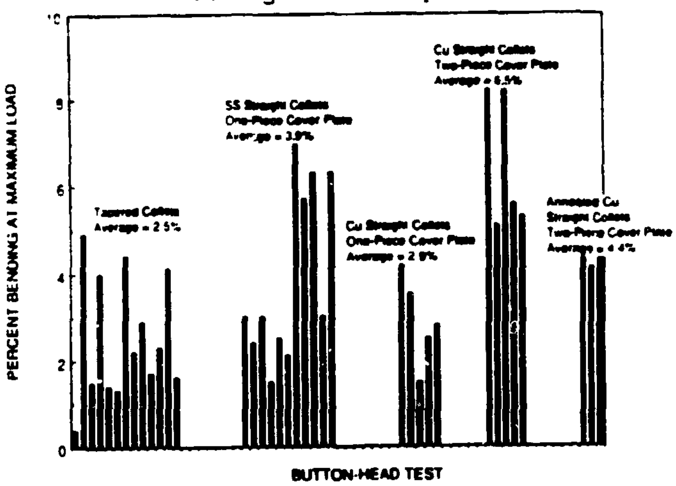

Figure 14. Percent Bending at Failure of the Gripped Area for Tensile Tests of Straight-shank Specimens (32.33).

However, the average load at failure for the annealed-copper, straight collets did approach that of the tapered collet system although the average percent bending did not show similar behaviour. It has been suggested (25) that the plastic deformation of the "soft" annealed copper produces a condition in the button-head similar to that of a hydrostatic pressure. Therefore, from the previous discussion, the induced stress concentration for this straight collet configuration would be expected to be similar to that of the preloaded, tapered collet system. The similarity of the stress concentrations is indicated by the similarity of the experimental test loads at failure as displayed in Figure 13. However, the test results also show [Figure 14] that while the plastic deformation allowed the straight collet system to sustain higher loads in the button-head by reducing the stress concentration, the deformation also may have introduced eccentricities which increased the percent bending.

CONCLUSIONS

To date, the following conclusions may be made: 1) FEA modeling has shown a nearly uniform, uniaxial stress field in the gage section although stress concentrations exist at the button-head and at the transition from the gage section to the shank. 2) friction at the collet/specimen interface may not be as critical as the inherent stress concentration in the button-head area which may be an unacceptable part of the straight collet gripping system. 3) good machining practices and proper dimensional checks are essential for successful and meaningful tensile test results. 4) the tapered collet system is able to sustain higher loads at lower percent bending errors than most configurations of the straight collet system.

REFERENCES

1. Cuccio, J., "Life Prediction Methodology for Ceramic Components in Advanced Engines," WBS Element 3.2.2.3. Ceramic Technology for Advanced Heat Engines. Oak Ridge National Laboratory. Oak Ridge, Tennessee. 1989.
2. Vaccari, D.L., Khandelwal, P.K., "Life Prediction Methodology for Ceramic Components in Advanced Vehicular Engines," WBS Element 3.2.2.2. Ibid.
3. Rudnick, A.., Marschall, C.W., Duckworth, W.H., Emrich, B.R., "The Evaluation and Interpretation of Mechanical Properties of Brittle Materials," AFML-TR-67-316, 1968.
4. Jones, M.H., Brown, W.F., Jr., "An Axial Loading Creep Machine," ASTM Bulletin, No. 211, 1956, pp. 53-60.
5. Seshadri, S.G., Chia, K-Y, "Tensile Testing Ceramics," Journal of the American Ceramic Society, Vol. 70, No. 10, 1987, pp. C242-C244.
6. Ohji, T.," Towards Routine Tensile Testing," Journal of High Technology Ceramics, Vol. 4, 1988, pp. 211-225.
7. Vaidayanathan, R., Sankar, J., Avva, V.S., "Uniaxial Tensile Characteristics of Silicon Nitride at Room Temperature," Ceramic E ne Science Proceedings, Vol. 9, No. 9-10, 1988, pp. 1383-1392.
8. Liu, K.C., Brinkman, C.R., "Tensile Cyclic Fatigue of Structural Ceramics," Proceedings of the 23rd Automotive Technology Development Contractors' Coordination Meeting, Dearborn, Michigan, October 21-24, 1985, P-165, SAE, Warrendale, Pennsylvania, 1986, pp 279-284.
9. "Investigation of High Temperature Tensile Testing Procedures", in The Report of Research and Investigation on Standardization of High Performance Ceramics, Asahi Glass Co., Ltd., IHI Co. Ltd., NGK Insulator Co. Ltd., Japan, 1986.
10. Fujita, H., Kawai, M., Takashi, H., Abe, H., Nakayama, J., "Uniaxial Tensile Fatigue Testing of Sintered Silicon Carbide under Cyclic Temperature Change," Fracture Mechanics of Ceramics, Volume 7, R.C. Bradt, A.G. Evans, D.P.H. Hasselman, F.F. Lange, eds. Plenum

Press. New York. 1987. pp 379-390.
11. Masuda. M.. Soma. T.. Matsui. M.. Oda. I. "Cyclic Fatigue of Sintered Si3N4." Ceramic Engine Science Proceedings. Vol. 9. No. 9-10. 1988. pp. 13871-1382.
12. Penny. R.K.. Ellison. E.G.. Webster. G.A.. "Specimen Alignment and Strain Measurement in Axial Creep Tests." Materials Research and Standards. Vol. 6. No. 1. 1966. pp. 76-89.
13. Smith. E.M.. Grant. C.. Booth. J.M.. "Equipment of Creep Testing at Variable Load and Temperature." Journal of Strain Analysis. Vol. 5. No. 2. 1970. pp. 145-154.
14. Grant. C.. "Axiality of Loading in the Tensile Test." Journal of Strain Analysis. Vol. 7. No. 4. 1972. pp. 261-265.
15. Jones. M.H.. Brown. W.F. Jr.. "Note on Performance of Tapered Grip Tensile Loading Devices." Journal of Testing and Evaluation. Vol. 3. No. 3. 1975. pp. 179-181.
16. Kossowsky. R.K.. Miller. D.G.. Diaz. E.S.. "Tensile and Creep Rupture of Hot-Pressed Silicon Nitride." Journal of Materials Science. Vol. 10. 1975. pp. 983-997.
17. Lange. F.F.. Diaz. E.S.. "Powder-Cushion Gripping to Promote Good Alignment in Tensile Testing." Journal of Testing and Evaluation. Vol. 6. No. 5. 1978. pp. 320-323.
18. Christ. B.W.. Swanson. S.R.. "Alignment Problems in the Tensile Test." Journal of Testing and Evaluation. Vol. 4. No. 6. 1979. pp. 405-417.
19. Lange. F.F.. Diaz. E.S.. Anderson. C.A.. "Tensile Creep Testing of Improved Silicon Nitride." Journal of the American Ceramic Society. Vol. 58. 1979. pp. 845-848.
20. Govila. R.K.. "Uniaxial Tensile and Flexural Stress Rupture Strength of Hot-Pressed Silicon Nitride." Journal of the American Ceramic Society. Vol. 65. 1982. pp.15-21.
21. Govila. R.K.. "High Temperature Uniaxial Tensile Stress Rupture Strength of Sintered Alpha SiC." Journal of Materials Science. Vol. 18. 1983. pp. 1967-1976.
22. Carroll. D.F.. Wiederhorn. S.M.. Roberts. D.E.. "Technique for Tensile Creep Testing of Ceramics." Journal of the American Ceramic Society. Vol 72. No. 9. 1989. pp. 1610-1614.
23. "Instron Super Grip." Instron Corporation. Canton. Massachusetts. PS-6-1202. 1988.
24. Carruthers. D.. Cuccio. J.. "Button-head Failures Necessitate Tensile Specimen Redesign." presented at Bi-annual Advanced Turbine Technology Applications Project Meeting. Garrett Auxiliary Power Division. Phoenix. Arizona. March 29. 1989.
25. Hecht. N.L.. "Environmental Effects in Toughened Ceramics." W.B.S. Element 3.3.1.4. University of Dayton Research Institute. Ceramic Technology for Advanced Heat Engines Project. Oak Ridge National Laboratory. Oak Ridge. Tennessee. 1989.
26. Cosmos/M 1.52A. Structural Research & Analysis Corp..Santa Monica. California. 1989.
27. Timonshenko. S.P.. Goodier. J.N.. Theory of Elasticity. McGraw-Hill Book Co.. New York. 1971. pp. 409-420.
28. ANSYS Engineering System. Revision 4.5. Swanson Analysis Systems Inc.. Houston. Pennsylvania. 1989.
29. SuperSap PC FEA. Algor. Pittsburgh. Pennsylvania. 1989.
30. Peterson. R.E.. Stress Concentration Design Factors. John Wiley & Sons. Inc.. New York. 1953.
31. American Society for Testing and Materials. "Sharp-Notch Tension Testing with Cylindrical Specimens." E 602-81. 1989 Annual Book of ASTM Standards. Vol. 03.01. 1989. pp. 587-592.
32. Jenkins. M.G.. Ferber. M.K.. Martin. R.L.. Poteat. R.M.. Tennery. V.J.. "Numerical and Empirical Verification of the Stress State in a Ceramic. Buttonhead. Tensile Specimen." Proceedings of the 27th Automotive Technology Development Contractors' Meeting. Dearborn. Michigan. October 23-26. 1989. Society of Automotive Engineers. Warrendale. Pennsylvania. 1990.
33. Jenkins. M.G.. Ferber. M.K.. Martin. R.L.. Tennery. V.J.. "Evaluation of the Stress State in a Buttonhead. Tensile Specimen for Ceramics." to be published in Proceedings of the 14th Annual Conference on Composites and Advanced Ceramics. Cocoa Beach. Florida. January 14-17. 1990. The American Ceramic Society. Westerville. Ohio. 1990.